

D R A

MANIPULATION OF PARTICLES

BY WEAK FORCES

IN ZERO G ENVIRONMENT

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A REPORT ON A STUDY CARRIED OUT FOR THE
SPACE SYSTEMS PRODUCTS DEPARTMENT OF GENERAL ELECTRIC

By

Dr. M.S. Adler, Dr. S.D. Savkar and H.R. Summerhayes

Physics & Electrical Engineering Laboratory
Corporate Research & Development
General Electric Company
Schenectady, New York

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I. INTRODUCTION

The work described herein is in support of a contract called "Beneficial Uses of Space" which the General Electric Company is carrying out for the National Aeronautics and Space Administration at this time.

The objective of the study described in this report is to develop quantitative relations between various force fields and their effects on the motion of particles of various sizes and physical characteristics. More particularly, the study is concerned primarily with weak force fields whose effects are ordinarily masked out by the large gravitational force field in the earthbound environment.

In a satellite, however, where the gravitational field is nullified by centrifugal force, these relatively weak forces are strong enough to produce substantial particle motions which should be useful in sorting or separating or otherwise manipulating a variety of particle types.

If the particles to be manipulated are floating in an evacuated chamber, their acceleration will be the important parameter, and we have found that very substantial accelerations can be achieved in many cases. On the other hand, if the particles are confined in a container filled with a viscous medium such as air or water, they will ordinarily reach a drag limited constant velocity quite quickly in response to the applied force field, so that velocity rather than initial acceleration is the significant parameter.

As shown in the body of the report, the particle accelerations produced in a vacuum or the particle velocities produced in a viscous medium are dependent on the nature and intensity of the force field, the viscous medium, if used, and the characteristics of the particles themselves. These dependencies offer a variety of potential means of sorting or separating mixed assemblages of particles. Combinations may be chosen which will produce motions primarily sensitive either to electrical properties, magnetic properties, size, or combinations of the above parameters with density, depending on the nature of the force field and medium used.

It is hoped that the general expressions developed for these particle force field interactions and the numerical examples worked out for typical cases will enable potential users of the space environment to identify fruitful applications in the manufacture or processing of various commodities.

It was not a part of the study reported here to seek out the applications, but rather to produce a "box of tools" useful in the search for applications.

The phenomena which have been explored for use in manipulating particles are as follows:

- LIGHT
- HEAT
- MICROWAVES
- SOUND PRESSURE
- PARTICULATE RADIATION
- ELECTROSTATIC FORCES
- MAGNETIC FORCES

Brief Description and Comparison of Forces

It is seen that each of the first three phenomena is a form of electromagnetic radiation. The net DC force produced on particles in the field of an electromagnetic wave results from the radiation pressure of the wave. These three forms of radiation, plus X-rays and gamma rays, which are also forms of electromagnetic radiation, are treated together in a single section on electromagnetic radiation effects. The force produced is proportional to the effective area of the particles times the intensity of the radiation.

Unlike electromagnetic waves, sound waves require a medium for transmission. Like electromagnetic waves, sound waves also produce a radiation pressure force which is proportional to the effective area of the particle and the intensity of the wave. In most cases, however, the particle is much smaller than the wavelength, and in this regime the effective size of the particle as far as scattering of the wave is concerned is much smaller than its actual size, so that very little force is produced. This is discussed further in the section on sound pressure, where another effect known as acoustic streaming, which may have more useful applications in particle manipulation is also discussed.

In the section on particulate radiation, the case of an electron beam impinging on particles is discussed. As in electromagnetic pressure and sound pressure, the force produced is proportional to the particle area and to parameters related to the beam intensity. Electron beams differ, however, from electromagnetic waves and sound waves in that the effective cross section area of the particle will generally be approximately equal to its actual cross section area as far as variations in particle size are concerned, so that the equations of motion would be expected to hold down to very small particles.

The report concludes with discussions of magnetic forces and electrostatic forces. We are all familiar with the fact that large forces are developed on soft iron parts in relays and solenoids by relatively small electromagnets, but it is not so commonly realized that nearly all substances have magnetic permeabilities differing sufficiently from unity to produce appreciable motions in a magnetic field in a zero gravity environment. The equations for magnetic forces and their resulting accelerations in a vacuum or velocities in viscous media are given, as well as numerical examples for various substances.

The electrostatic force of dielectrophoresis is shown to be exactly similar in form to the magnetic force. It depends on the strength of the electric field and its degree of non-uniformity, the dielectric constant of the particle, and the volume of the particle, in exactly the same functional way that the magnetic force depends on the strength and the non-uniformity of the magnetic field, the magnetic permeability of the particle, and the volume of the particle.

These electrostatic and magnetic forces are dependent on particle volume rather than on cross sectional area as in the case of the forces arising from electromagnetic waves, sound waves and particulate beams. They, therefore, furnish a sensitivity to different particle parameters, a fact which adds versatility to the application possibilities.

Throughout this report we refer to the motion of particles of micron size in an evacuated or gas filled chamber resulting from various applied forces. One must realize that this type of interaction would in general be totally swamped out in the earth's gravitational field, since the particles would all fall to the bottom of the chamber, particularly in the case of a vacuum chamber. In the zero gravity environment, however, the particles would be floating lazily about the chamber, and the applied force would be effective in sorting or other manipulative processes.

For the case of particles such as viruses which must be preserved in a water solution, the effects of the forces studied here are very small indeed because of the drag force exerted on the particles by the viscosity of the liquid medium. For example, the summary chart on page 39 lists limiting velocities for particles in air as the viscous medium for various force fields. Limiting velocities ranging from 10^{-5} to 3×10^{-3} cm/sec are shown for typical forces and particle sizes in air at atmospheric pressure and temperature. But in a water solution which has viscosity fifty times greater than air, these velocities are reduced by fifty fold. In electrophoresis systems, separation times of tens of minutes in liquid colloid solutions are not uncommon, corresponding to velocities of the order of 10^{-3} cm/sec, but the predicted values for the velocities in liquid solutions for the forces described herein would be generally much lower.

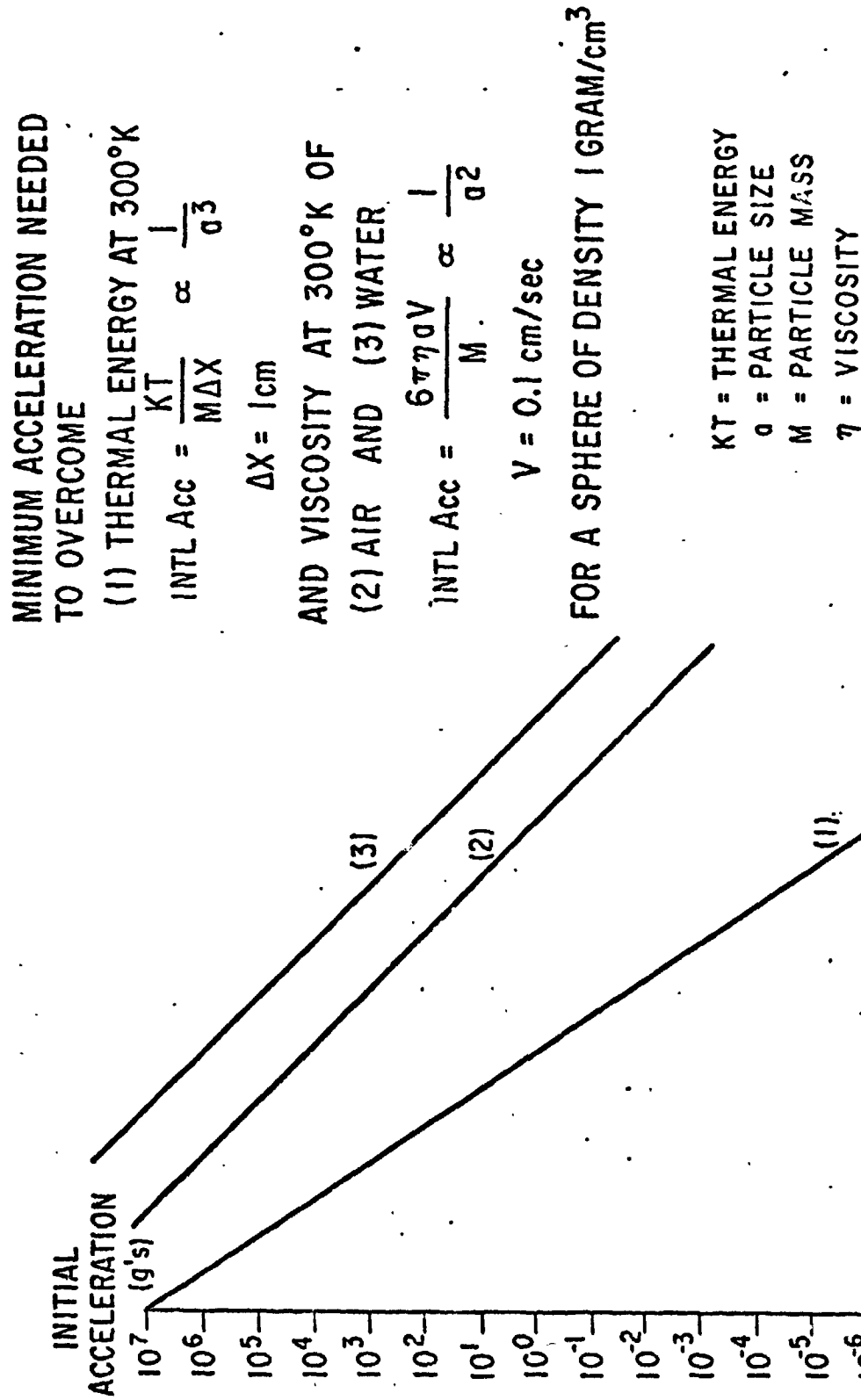
II. THERMAL MOTION AND VISCOUS FORCES

Before actually describing the details of the five categories of small forces that were considered in the study, it is worthwhile to discuss the environmental constraints that will apply in general to all of the forces. The two issues to be considered are (1) the competing effects of thermal motion and (2) the viscous forces that will be acting on the particles if they are in fluid. A particle at absolute temperature T will have a thermal kinetic energy $\langle \frac{1}{2} M v^2 \rangle = \frac{3}{2} kT$, where k is Boltzmann's constant, M is the particle mass, v is the particle velocity, and the brackets indicate an average. If a gas of these particles is in a container and a force F is applied in a direction perpendicular to a wall of the box, the equilibrium distribution of the particles is proportional to $\exp(Fx/kT)$ where x is the distance away from the wall. Approximately 10% of the particles will be located a distance Δx away from the wall, much like the fact that 70% of the earth's atmosphere is located within 10 km of the earth's surface due to the force of gravity. It should be noted that this is an equilibrium distribution (the Maxwell-Boltzmann distribution) and does not contain any information as to the time necessary to establish the equilibrium. This time might, in fact, be very long if the medium were a viscous fluid. In Figure II-1, the initial acceleration needed at 300°K to establish a distribution with $\Delta x = 1 \text{ cm}$ is plotted (curve (1)) as a function of particle size for spherical particles with densities of one gram/cm³. The ordinate of Fig. II-1 is the initial acceleration that a particle would experience in a viscous fluid as a result of the applied force, and is the same as the particle acceleration in a vacuum, F/M , where M is the particle mass. It is used here because much of the subsequent work is given in terms of the acceleration of particles in a vacuum rather than the force on the particles. As can be seen, the necessary initial acceleration is a function of the reciprocal of the particle radius cubed.

In many cases the particles of interest will be imbedded in a viscous fluid or gas and it is then likely that the need for a reasonable particle velocity will determine the necessary force rather than the thermal considerations given above. This can also be seen in Fig. II-1 where Stokes law² is used to indicate, as a function of particle radius, the initial acceleration needed to achieve a final velocity of 0.1 cm/sec for spherical particles of density 1 gram/cm³ at 300°K for an air medium at standard pressure (curve (2)) and for a water medium (curve (3)). As can be seen, both of these curves indicate a much higher needed initial acceleration than given from the thermal considerations. Also, as can be seen, this requirement on initial acceleration varies only as the reciprocal of the radius squared. These curves are meant to serve only as an illustration of the type of dependencies to be expected. For a specific application, the curves may require scaling since both will depend on temperature and curve (2) depends strongly on pressure.

It should be noted that the restriction of particle velocity in a viscous fluid is not necessarily a detrimental effect. The dependence of drift velocity in a viscous fluid on particle radius and force, $v = 6\pi\eta R F$ (see Fig. II-1 for explanation) may actually become an additional means for achieving the separation of particles of different sizes if forces of sufficient magnitude can be achieved.

Figure II-1



III. RADIATION PRESSURE

In this section, we will consider the use of radiation pressure as a means for moving small particles and also as a means for achieving selective separation of small particles on the basis of size of the particles, refractive index of the particles, and the wavelength of the electromagnetic radiation. We will first derive an equation for the acceleration in a vacuum produced by radiation pressure. We will then consider the entire range of electromagnetic radiation from microwaves to x-rays and gamma rays and attempt to characterize the size of achievable acceleration as a function of wavelength in order to determine the most promising sources of electromagnetic radiation for radiation pressure. Anticipating the result of this study, we will focus attention on the radiation pressure produced by sources in the near ultraviolet, visible, and infrared wavelength range. Resonant excitation of atomic absorption lines in atoms and molecules will also be treated.

A. General

The radiation pressure of an electromagnetic wave on a body that is perfectly absorbing is simply W/c ,⁽³⁾ where W is the power carried by the wave per unit area and c is the speed of light. Expressed another way, W/c is the rate that momentum can be removed from the wave per unit area of the wave. The force on a body is given, in general, as

$$F = \frac{W}{c} \cdot Q_{pr} \cdot A \quad \text{III-1}$$

where Q_{pr} is the efficiency factor for radiation pressure and A is the cross sectional area of the body. Consequently, $Q_{pr} \cdot A$ is defined as the cross-sectional area of the body that is effective in receiving momentum from the electromagnetic wave. Q_{pr} contains the "physics" of the momentum transfer process and, in general, is a complicated function of the parameter $\alpha = 2\pi a/\lambda$ and the refractive index n of the body, where a is related to the size of the body and λ is the wavelength of the incoming radiation. Q_{pr} can be calculated from the efficiency factors for absorption Q_{abs} and for scattering Q_{sca} and is given as

$$Q_{pr} = Q_{abs} + Q_{sca} \cdot \langle 1 - \cos \theta \rangle \quad \text{III-2}$$

where θ is the angle between the scattered light and the transmitted light. This equation indicates that all the radiation that is absorbed contributes directly to momentum transfer, but scattered radiation only contributes if it is scattered away from the forward direction. For example, perfectly absorbing particles that are large compared to the wavelength of the incoming radiation (the geometrical optics requirement) have an effective cross sectional area equal to their geometrical cross section i.e. $Q_{pr}=1$. This is analogous to an inelastic collision in mechanics. A large perfectly reflecting particle has $Q_{pr}=2$, so that the effective cross section for momentum transfer is twice

the geometrical cross section, which is analogous to an elastic collision in mechanics.

The initial acceleration in a vacuum for spherical particles of radius a (cm) and density ρ (grams/cm³) in a radiation field of intensity W (watts/cm²) is given by

$$A_{cc} = \frac{F}{M} = 2.5 \times 10^{-4} \frac{Q_{pr} W}{\rho a} \text{ cm/sec}^2 \quad \text{III-3}$$

where M is the mass of the particle. Although the details of the subsequent work are for spherical particles since they are easier to handle analytically, the results can be expected to approximately apply to particles of irregular shapes by making some appropriate estimate as to an average radius.

B. Survey of Radiation Pressure over a Broad Spectrum of Wavelengths

In Table III-1, we have characterized the size of acceleration achievable from a broad spectrum of electromagnetic radiation sources. To do this, we have made use of the fact that maximum acceleration is achieved with $\chi = 2\pi a/\lambda \approx 1$, and for this value of χ , the efficiency factor for strongly refracting and absorbing materials ($n, k > 1$) is of the order of unity. The power density was chosen as $W = 1 \text{ watt/cm}^2$ and the particle density as $\rho = 1 \text{ gram/cm}^3$.

Table III-1

Type of radiation	Frequency Range or Typical Frequency	Nominal Wavelength	Min. Part. Dia. for Good Q_{pr}	Acceleration in a vacuum of the min. particle in a 1 watt/cm ² radiation field
Microwave, X band	8 - 12.5 GHz	3 cm	0.75 cm	0.00067 cm/sec ²
Microwave, K band	18 - 26.5 GHz	1 cm	0.25 cm	0.002 cm/sec ²
Microwave, O band	60 - 120 GHz	.3 cm	0.075 cm	0.0067 cm/sec ²
* Infrared, submillimeter	$1.2 \times 10^{11} \text{ Hz to } 10^{13} \text{ Hz}$	2500 μ to 30 μ	630 μ to 7.5 μ	.0079 cm/sec ² to .67 cm/sec ²
Infrared, CO ₂ laser	$2.83 \times 10^{13} \text{ Hz}$	10.6 μ	2.6 μ	1.9 cm/sec ²
Visible light	$6 \times 10^{14} \text{ Hz}$	0.5 μ	0.12 μ	40 cm/sec ²
Ultra Violet	$6 \times 10^{15} \text{ Hz}$	0.05 μ	0.012 μ	400 cm/sec ²
* X-rays and γ rays	$10^{17} \text{ to } 10^{22} \text{ Hz}$	30 \AA to 0.0003 \AA	1.0 \AA (atoms)	.0026 cm/sec ²

* Note that the power density in these regions which is currently available from state-of-the-art power sources is generally much less than 1 watt/cm² so that in practical equipment the accelerations achievable in the asterisked rows would be proportionally less than the values shown. X-ray power is about 10⁻² watts/cm² as shown on page 8, and except for a few experimental lasers at a few hundred microns, there are no good sources in the submillimeter region.

It is evident from the table that even the highest microwave radar frequencies can in general produce only very small accelerations on the smallest particles with which they can interact efficiently. The acceleration values can be bettered somewhat if a higher intensity than the nominal C.W. power intensity of 1 watt/cm² can be achieved either by use of higher power sources or by focussing of the energy.

The submillimeter infrared region might produce usable accelerations if reasonable power levels were available, but the means of generating and guiding submillimeter waves are very limited, and the power levels available decrease rapidly as wavelength decreases in this region.

It would seem that because of their short wavelength X-rays would be a good source of accelerating individual atoms and molecules because of the favorable value of the parameter χ . However, the high energy of the X-rays makes them very penetrating and consequently they do not interact strongly with individual atoms and molecules. The lead atom, which has relatively a very large X-ray cross section, has a cross section for emission of photoelectrons (the largest) of only 2.7×10^{-20} cm² for 10 KeV X-rays ($\lambda = 1.24 \text{ \AA}$). This corresponds to an efficiency factor of only $Q_{pr} = 10^{-4}$. Other atoms with smaller atomic numbers have correspondingly smaller effective cross sections. For example, the air molecule has a cross section two orders of magnitude smaller than lead.⁵ Furthermore, the available power levels for X-ray sources are also very small. The 1 watt/cm² value used to compute the accelerations in the above table is probably at least two orders of magnitude larger intensity than can be conveniently achieved. For example, the Coolidge tube, long the standard source of X-rays, produces only 3.9×10^{-5} watts/cm² at a distance of one meter or 3.9×10^{-3} watts/cm² at 10 cm from the target with an accelerating potential of 100 keV and a current of 10 ma.⁶ Therefore, the acceleration of 0.026 cm/sec² in the table represents a value that probably could never be achieved. In addition, inspection of Fig. II-1 reveals that based on thermal energy arguments alone the minimum acceleration needed for gas molecules is approximately 10^9 cm/sec². Consequently, we are well justified in not pursuing the issue of x-ray sources further.

C. Radiation Pressure from Sources in the Near-Ultraviolet, Visible & Infrared Region Non Absorptive Materials

Having focussed attention on the range of wavelengths near the visible, we will now show the results of calculations of particle acceleration due to radiation pressure as a function of particle size, refractive index, and wavelength of the source radiation. For each set of particle parameters considered, the calculations were done for three different wavelengths of source radiation, each corresponding to a known laser frequency: (1) .325 μ (HeCd), (2) .6328 μ (HeNe), (3) 10.6 μ (CO₂). The calculations were made in two parts, the first corresponding to nonabsorbing particles (real refractive index) and the second to absorbing particles (complex refractive index).

As noted earlier, the process of calculating the acceleration due to radiation pressure amounts to the determination of the efficiency factor for radiation pressure Q_{pr} . In general, for a sphere, this amounts to solving Maxwell's equations where one expresses the incoming wave, the wave inside of the sphere, and the outgoing wave in terms of an infinite sum of spherical Bessel functions and then matches boundary conditions at the surface of the sphere. This was first done by Gustav Mie in 1908.⁷ For the work to be presented here, Q_{pr} was obtained by piecing together and convolving separate more restricted theories of scattering such as the theory of Rayleigh scattering⁸ and anomalous diffraction⁹ with series expansions¹⁰ and numerical calculations¹¹ derived directly from the formal Mie scattering theory.

The initial acceleration as a function of particle size for non-absorptive spheres with refractive indices of $m = 1.1, 1.3, 1.5, 1.8$ and 2 for the three above noted wavelengths is shown in Figs. III-1, -2, and -3. The particle density is $\rho = 1 \text{ gram/cm}^3$, and the radiation intensity is $W = 1 \text{ watt/cm}^2$. The dashed lines in these figures are a reproduction of the requirement which is shown in Fig. II-1 for the minimum initial acceleration necessary to confine approximately 70 percent of the particles (i.e. $1 - \frac{1}{e} \approx 2.718$) in a distance of 1 cm. It should also be noted that the acceleration is plotted in units of the earth's gravity ($G = 9.80 \times 10^2 \text{ cm/sec}^2$).

The most interesting feature of the results shown is the peak in the particle size dependence of the initial acceleration. For a given wavelength, the peak is approximately independent of refractive index but diminishes in size as $m-1$ approaches zero. The position of the peak occurs for a particle radius that satisfies the requirement that $x = 2\pi a/\lambda \approx 1$ and the maximum acceleration is approximately given by the following empirical formula

$$\frac{\text{Int'l Acc}}{G} = \frac{.0183}{\lambda} (1.4(m-1)^2 - 0.4(m-1)^3) \quad \text{III-4}$$

where λ in this expression is in microns and

where the refractive index is constrained to be in the range $1 \leq m \leq 2$. The particle radii for which the acceleration is $\geq .707$ of the maximum value (given above) is approximately given by the following empirical formula

$$(\lambda/2\pi - \Delta a/2) < a < (\lambda/2\pi + \Delta a/2) \quad \text{III-5}$$

where

$$\Delta a = 2\lambda/(m-1)$$

The accuracy of the above two formulae is ~ 10 percent. For particle radii such that $x < 0.5$ (eq. for $\lambda = .325\mu$, $a < .025\mu$; for $\lambda = .6328\mu$, $a < .05\mu$; for $\lambda = 10.6\mu$, $a < 0.8\mu$) the acceleration falls off as a^3 . The reason for this is that for these small values of x the scattering is essentially Rayleigh scattering⁸ so that Q_{pr} has a dependence of a^4 but the $1/a$ in Eqn. III-3 reduces this to the a^3 dependence mentioned above. For particle radii such that $x > 4$ (eq. for $\lambda = .325\mu$, $a > 0.2\mu$; for $\lambda = .6328\mu$, $a > .4\mu$; for $\lambda = 10.6\mu$, $a > 7\mu$) the acceleration falls off as $1/a$ because for these larger values of x Q_{pr} is approximately constant so that the $1/a$ dependence in Eqn. III-3 prevails.

Figure III-1

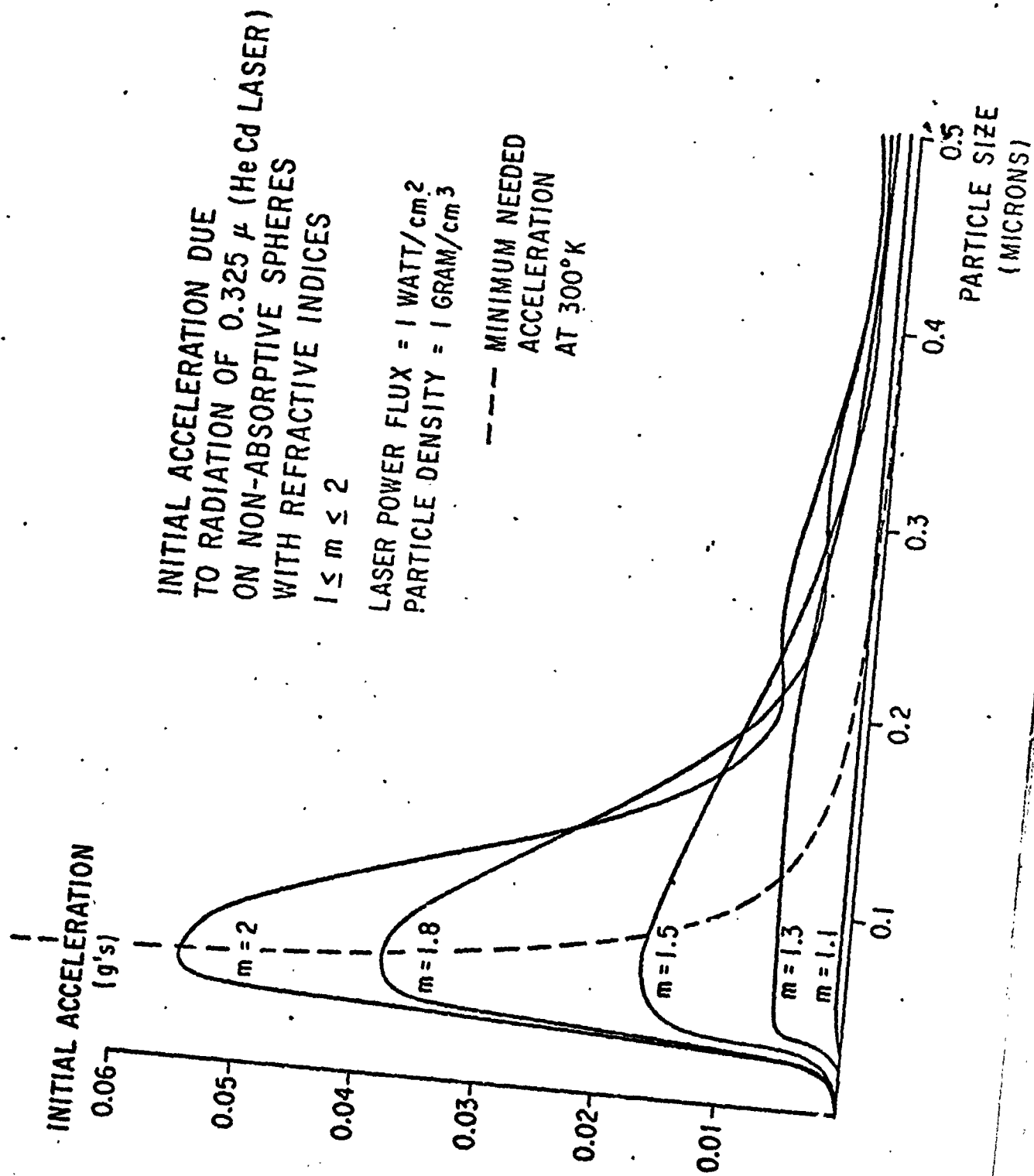


Figure III-2

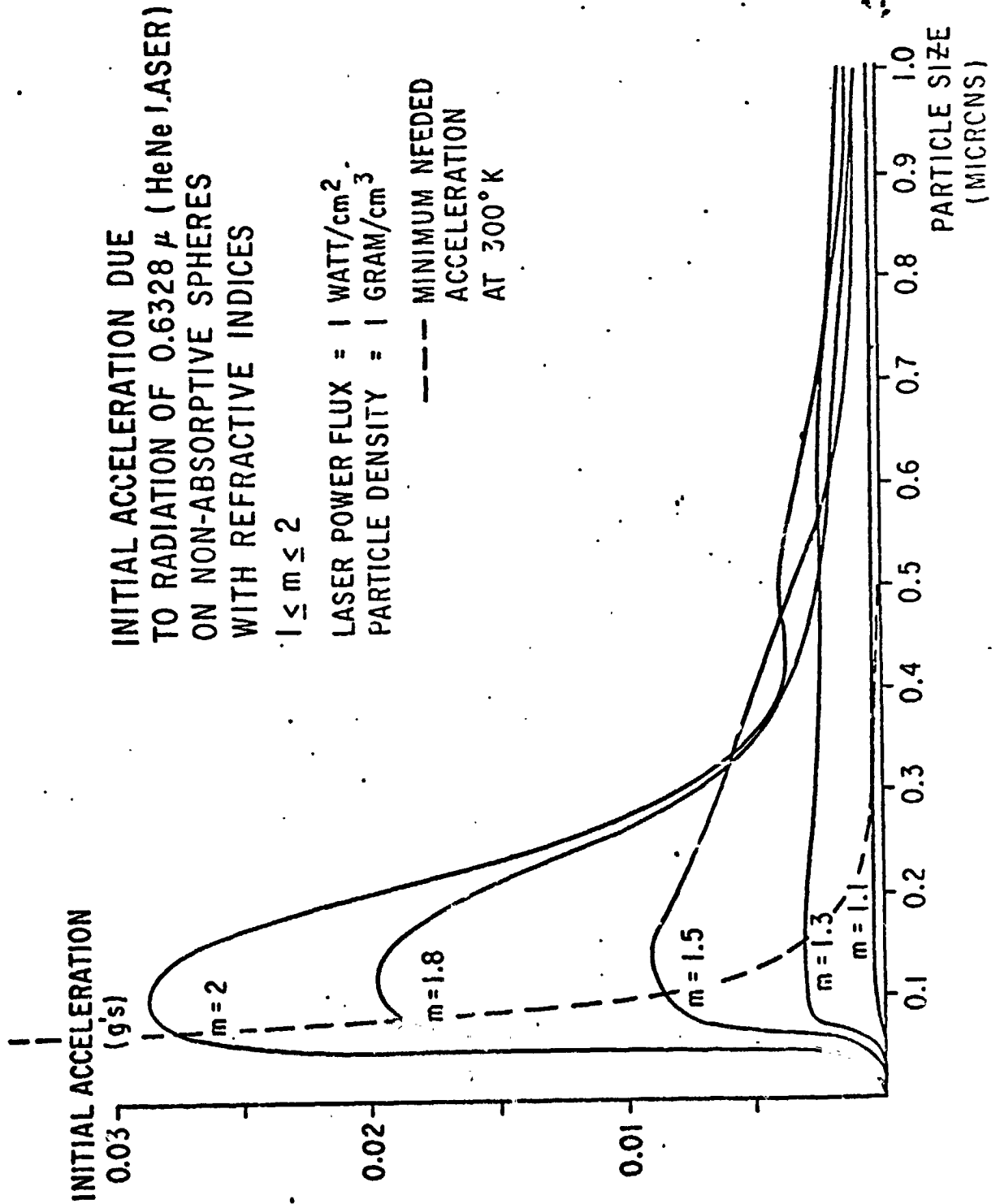
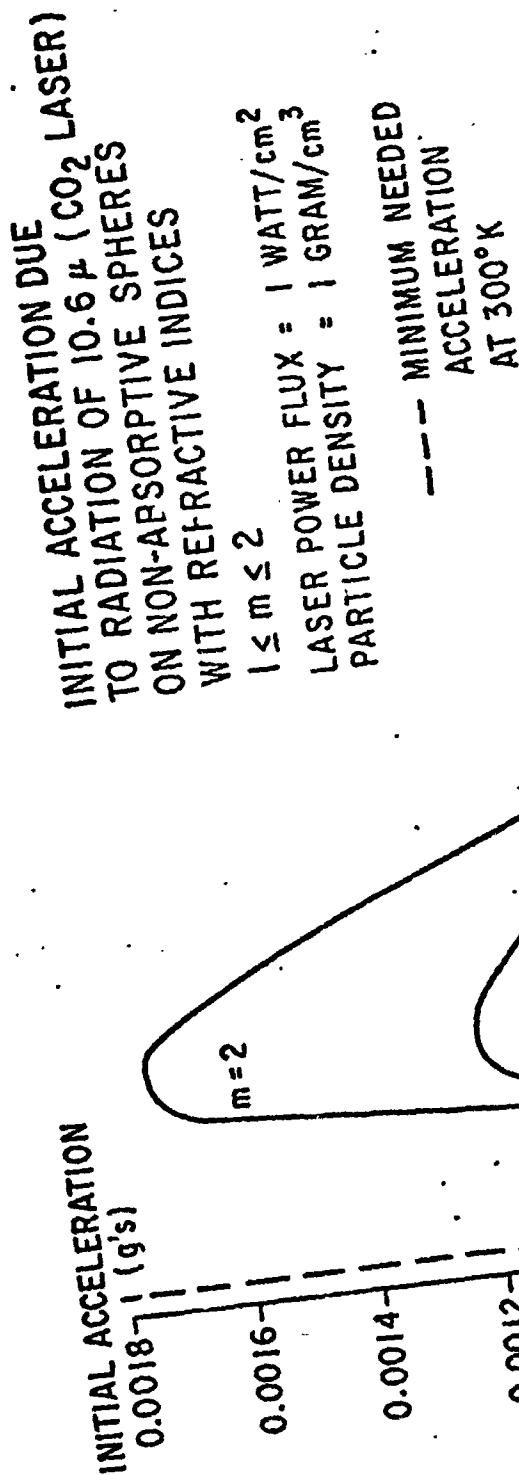


Figure III-3



The range of refractive indices $1 \leq m \leq 2$ is sufficient to encompass almost all non-absorptive materials. The refractive index for most substances can be found in the Handbook of Chemistry and Physics.¹² Refractive indices for some representative materials are given in Table III-2.

Table III-2

Ice	m= 1.31	
Water	m= 1.33	
Sulphur	m= 1.99	
Chlorine (liquid)	m= 1.385	
Hydrogen (liquid)	m= 1.11	
Aluminum oxide	m= 1.67	
Sodium Iodide	m= 1.77	
Sodium Chloride	m= 1.53	
Albite glass	m= 1.49	
Amber	m= 1.55	
Ivory	m= 1.54	
Quartz	m= 1.5	
Virus	m= 1.6	(a = 0.05 μ)
Protein	m= 1.6	(a = 0.1 μ)
Bacteria	m= 1.4	(a = 1.0 μ)

Although the refractive index of most non-absorptive materials is not a drastic function of wavelength in the region we are dealing with, there usually is some change ($\sim 30\%$) with wavelength, particularly in the infrared. The dependence of the refractive index on wavelength for a number of materials can be found in the American Institute of Physics Handbook.¹³

Absorptive Materials

The task of obtaining the efficiency factor for radiation pressure Q_{pr} is more difficult for absorptive materials because, except for small values of the parameter χ , there is no recourse other than the formal Mie theory. However, the task of carrying out a calculation using this complex theory is beyond the scope of this study. Consequently for the $\lambda = 0.325\mu$ and $\lambda = 0.6328\mu$ wavelengths we have only treated one refractive index, $m = 1.27 - 1.37i$ where $i = \sqrt{-1}$. This refractive index is actually that for iron at $\lambda = .4\mu$ and is one for which Q_{pr} has been calculated.¹⁴ It, however, is not precisely equal to the refractive index for iron at the specific wavelengths considered here (eq. for $\lambda = .325\mu$, $m = 1.16 - 1.27i$; and for $\lambda = .6328\mu$, $m = 1.7 - 1.84i$)^{12,15} but is typical of the refractive indices of many metals in this range of wavelengths, and thus serves as an illustration of the type of behavior to be expected. With this introduction, the initial acceleration as a function of particle size for the wavelength $\lambda = .325\mu$ and $\lambda = .6328\mu$ is shown in Fig. III-4. The dotted line in the figure is the initial acceleration needed to achieve a 1 cm concentration density of the type described earlier, but for this case the temperature is taken at 1000°K. This was done because 1000°K appears to be a more realistic operating temperature than 300°K since the materials are absorptive and will heat up when illuminated with high intensity radiation. The initial acceleration as it appears in the figure has been normalized with respect to the density of the particle material so that the figure can be readily applied to a variety of materials. To find the initial acceleration for a specific material, one must only divide the acceleration appearing in the figure by the specific gravity of the material.

In common with the previous results for non-absorptive spheres, there is a peak in the acceleration dependence for values of particle size where the parameter $\chi = 2\pi a/\lambda \cong 1$. Similarly, at larger particle radii, the acceleration falls off as $1/a$ since the efficiency factor for absorptive particles also becomes constant. The main difference between the absorptive and non-absorptive particles is that the acceleration does not fall off at small values of particle radius but approaches a constant value. This is due to the fact that while the efficiency factor for scattering diminishes as a^4 for small particles (i.e. $\chi < .5$), the efficiency factor for absorption diminishes only as a .⁽⁸⁾

The situation changes quite dramatically in the infrared region since the refractive index of metals becomes quite large and is dominated by the conductivity of the material. It is given for $\lambda > 5\mu$ as¹⁸

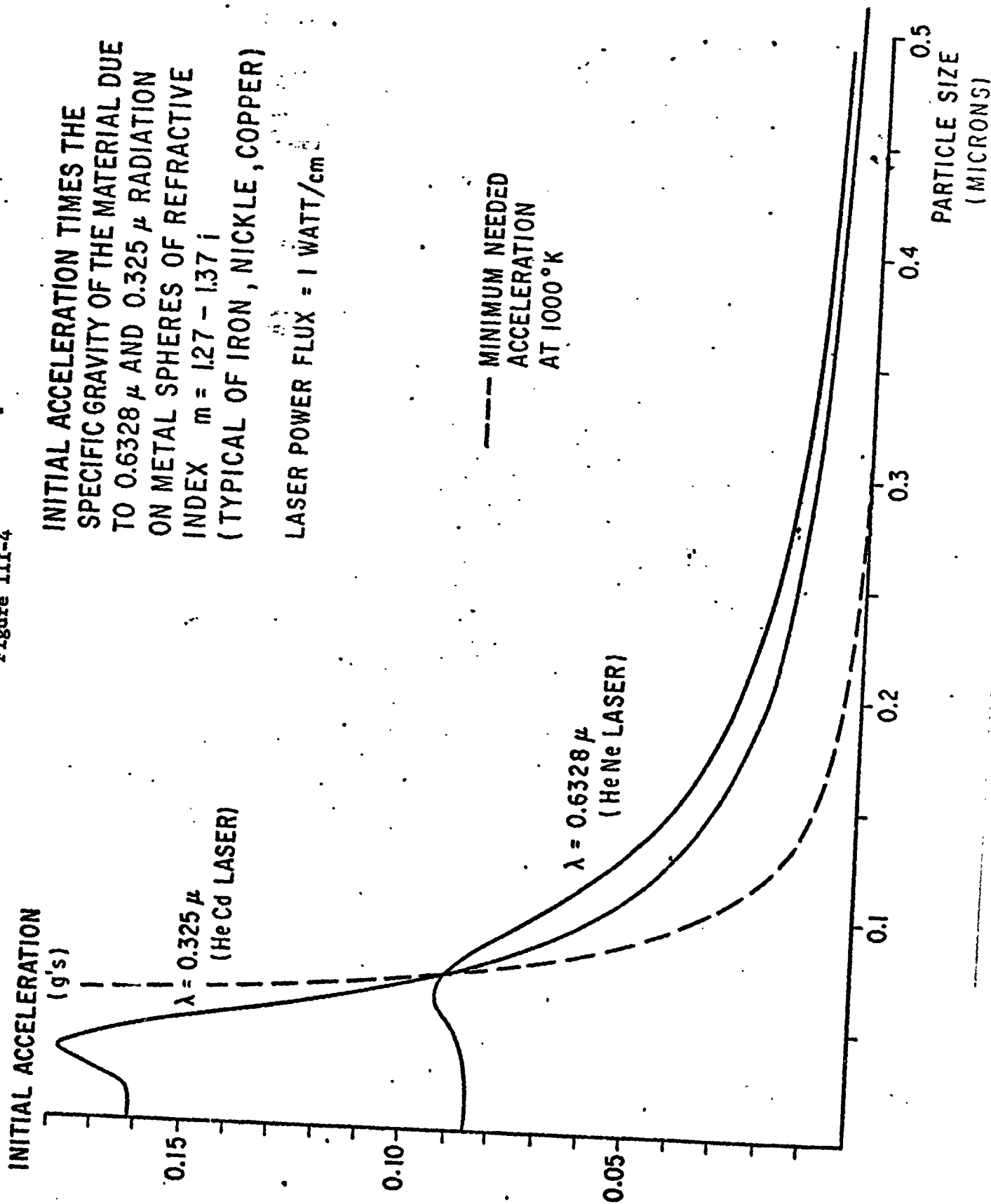
$$m = (30\sigma\lambda)^{1/2}(1-i)$$

III-6

where σ is the conductivity in MKS units and $i = \sqrt{-1}$. When $m\chi > 10$, the scattering dominates the absorption, and the spheres can be treated as perfectly reflecting for purposes of calculating the radiation pressure.

* The complex part of the refractive index is sometimes referred to as the absorption index, and reflects the fact that the material is conductive and thus can absorb energy.

Figure III-4



For this case, all metals (where $m\chi > 10$) will behave identically. This requirement that $m\chi > 10$ is essentially a requirement that the skin depth be small in comparison to the particle size. For the case of $m\chi < 1$ absorption dominates the scattering and the theory becomes more complicated with different metals behaving differently depending on their conductivity. The initial acceleration for the case of iron (but normalized with respect to density) at $\lambda = 10.6\mu$ is shown in Fig. III-5. As can be seen, there are two regions of interest, one dominated by scattering and the other by absorption. In the scattering region, the normalized acceleration shown would be identical to that for metals which have a conductivity at least as large as iron, since the efficiency factor for this portion of the curve was taken from that of perfectly reflecting spheres.¹⁹ The position of the peak in the scattering region also occurs for particle radii where $\chi = 2\pi a/\lambda \approx 1$. The region of the curve marked absorption is much more complicated and is unique to iron. In this region, the particle size is comparable or smaller than the skin depth. A complete discussion of this region is found in Van De Hulst.²⁰

An additional constraint is imposed in the case of absorptive particles, in that there is a limit to the amount of incoming radiation intensity that the particles can accept in order to avoid melting. We have calculated this power for iron spheres on the basis of equating the power absorbed from the radiation to the power lost to black body radiation¹⁶ at the melting temperature of iron ($\sim 1800^\circ K$). That is

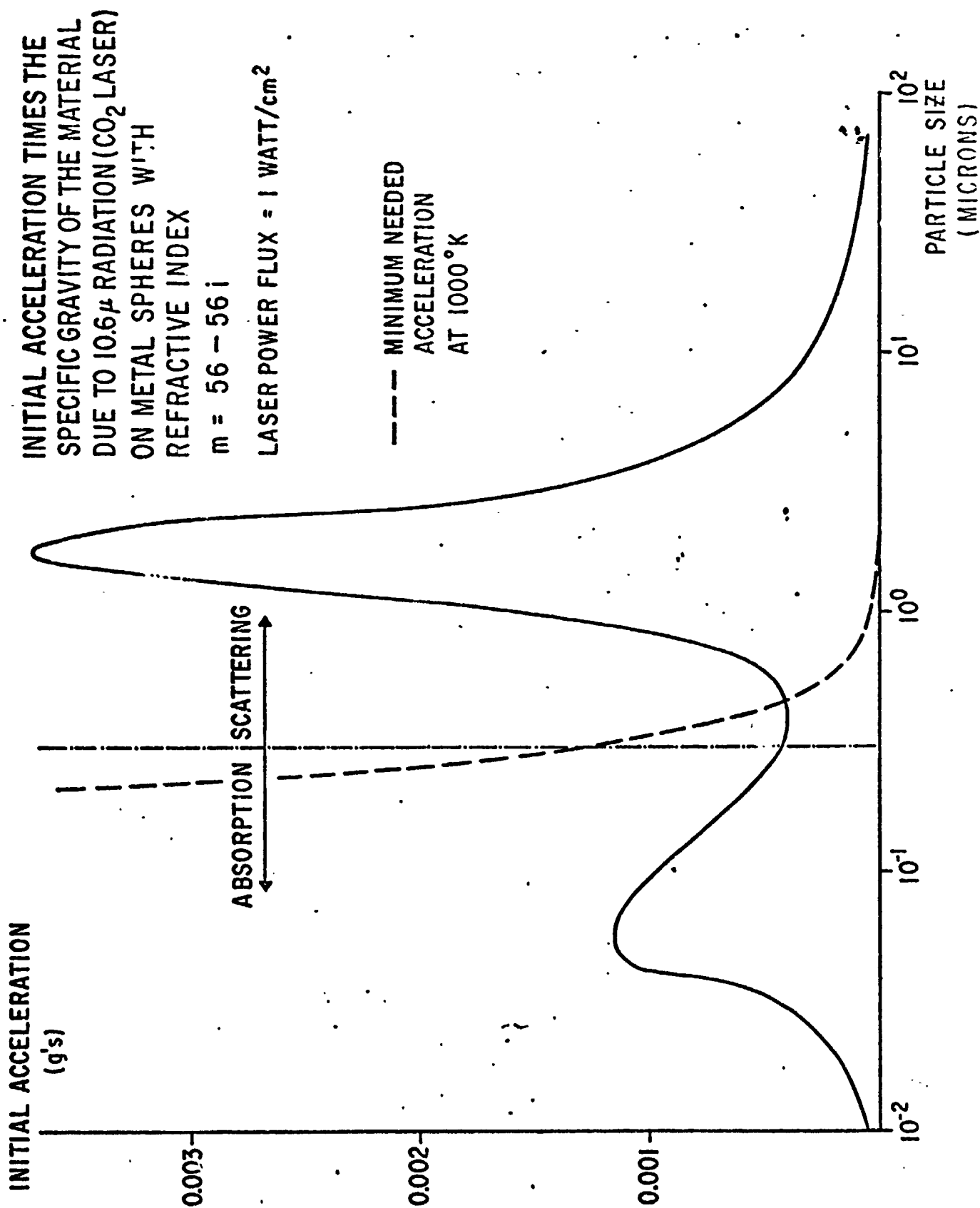
$$\text{Power lost} = Q_{abs}(\lambda_b) \cdot 4\pi a^2 \cdot \sigma T^4 \quad \text{III-7}$$

$$\text{Power absorbed} = Q_{abs}(\lambda) \cdot \pi a^2 \cdot W \quad \text{III-8}$$

where $\sigma = 5.67 \times 10^{-5} \text{ gm/sec deg}^4$ is the Stefan-Boltzmann constant, T is the temperature, $Q_{abs}(\lambda_b)$ is the efficiency factor for emission at the center of the range of wavelengths corresponding to black body radiation at temperature T , $Q_{abs}(\lambda)$ is the efficiency factor for absorption at the wavelength of the incident radiation, a is the particle radius, and W is the intensity of the incident radiation. In calculating the radiant power lost to black body radiation, Kirchhoff's law²⁰ states that the efficiency factor for emission (the emissivity) at a particular wavelength is equal to the efficiency factor for absorption at the same wavelength. This is the reason that the factor $Q_{abs}(\lambda_b)$ is included in Eqn. III-7 for the emissivity. With this explanation, the maximum incident radiation intensity in order to avoid melting iron spheres is plotted as a function of particle radius in Fig. III-6 for $\lambda = 3.25\mu$ and $\lambda = 6.325\mu$ and in Fig. III-7 for $\lambda = 10.6\mu$. These plots are superimposed on plots of the initial acceleration with the original power level of 1 watt/cm² taken from Fig. III-4 and Fig. III-5.

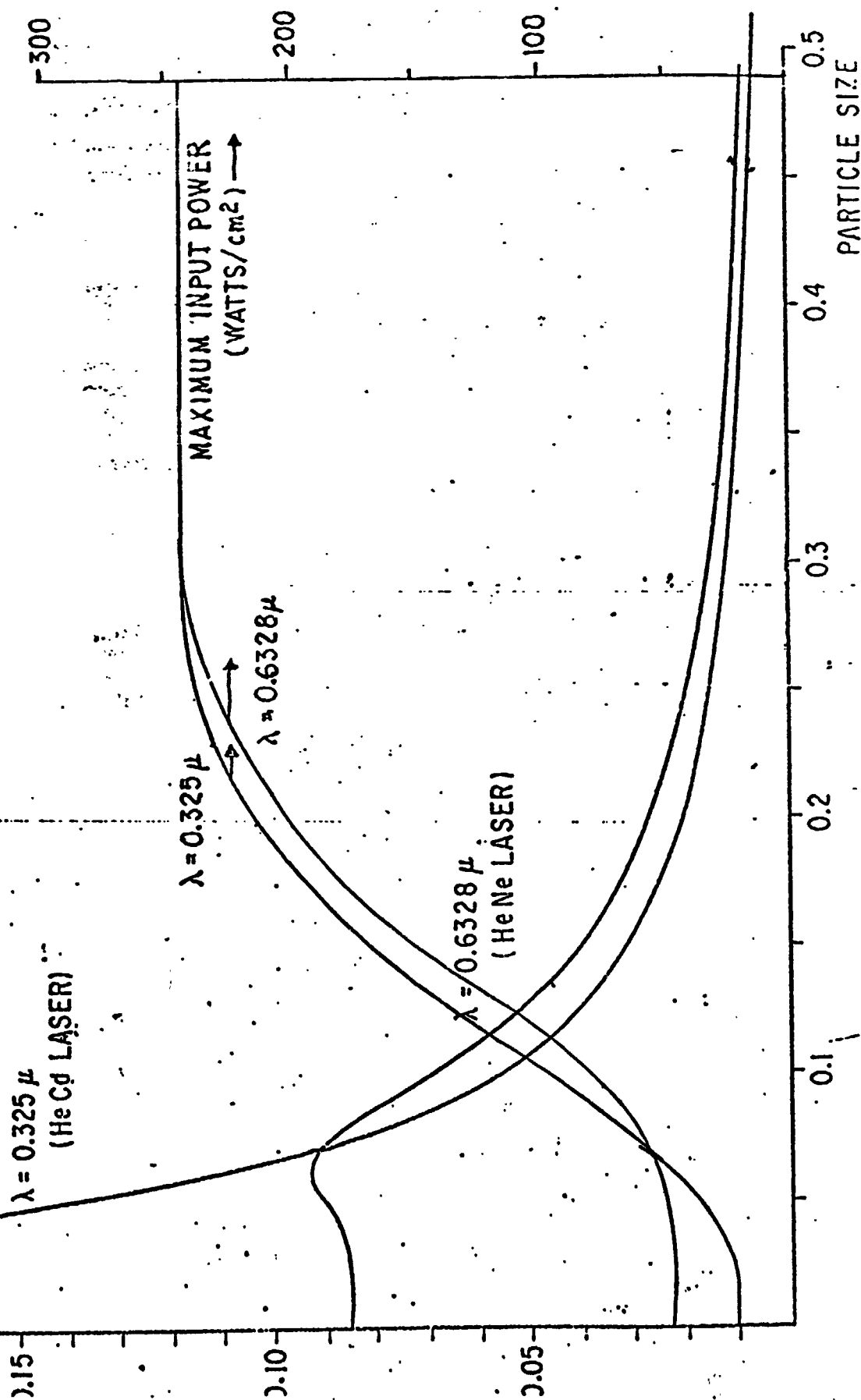
The dependence of the maximum power on particle size is due to the dependencies of $Q_{abs}(\lambda_b)$ and $Q_{abs}(\lambda)$. As can be seen, the initial acceleration can be greatly increased in all cases before melting will occur. The allowable power level is so high in the case of 10.6μ radiation that it is unlikely it will be approached in practice. The large size is due in part to the large size of

Figure III-5



INITIAL ACCELERATION
(g's)

FIGURE III-6
MAXIMUM INPUT POWER AT 0.325 μ AND
0.6328 μ TO AVOID MELTING IRON SPHERES



INITIAL ACCELERATION

(g's)

0.003

0.002

0.001

10⁻²

10⁻¹

10⁰

10¹

10²

ABSORPTION SCATTERING

MAXIMUM
INPUT POWER

(WATTS/cm²) →

30,000

20,000

10,000

PARTICLE SIZE

FIGURE III-7

MAXIMUM INPUT POWER AT 10.6μ

TO AVOID MELTING IRON SPHERES

the refractive index at 10.5μ , making the iron spheres seem highly reflecting, but it is also due to the fact that the black body radiation occurs at a much shorter wavelength ($\lambda \approx 1\mu$) which places it in a region where the efficiency factor for absorption, and hence emission, is much larger. The size of the allowable power levels (and the fact that high power levels can be achieved with the CO_2 laser) and the lower required initial accelerations for large particles (See Fig. II-1) more than compensate for the smaller accelerations obtained with the calculated power level of 1 watts/cm^2 seen in Fig. III-5 and Fig. III-3.

Resonant Radiation Pressure

The radiation pressure on atoms and molecules was also considered. However, the light scattering formalism as treated above for particles in the micron size range results in accelerations much too small when compared to thermal energies from atoms and molecules. Referring to Fig. II-1, it can be seen that to satisfy the requirement based on thermal energy (curve 1) for a particle with a radius of 3 \AA (atomic size) would require an initial acceleration of $\sim 10^5 \text{ G's}$. As will be seen below, accelerations on this order can be obtained from atomic resonant absorption in the visible wavelength range.

The force resulting from resonant absorption is given as ²¹

$$F = \frac{h}{\lambda} \cdot \tau_N^{-1} f \quad \text{III-9}$$

where h is Planck's constant, λ is the wavelength corresponding to resonant absorption, τ_N is the natural lifetime of the excited state, and f is the fraction of time the atom spends in the upper (excited) state. The quantity f is linearly dependent on radiation intensity at very low power levels but eventually stops increasing with increasing power since ultimately the atom can only absorb a photon until after returning to the ground state, a time delay which is equal to the lifetime of the excited state. At the point when the absorption does not increase with power level, the line is said to be saturated.

This equation has a simple interpretation. The quantity h/λ is the momentum of a photon and $\tau_N^{-1} f$ is the number of photons per second that the atom can absorb as limited by the lifetime of the excited state τ_N and the amount of time the atom spends in the ground state without absorbing a photon $(1 - f)$. The size of this force can be estimated by letting $\lambda = 5000 \text{ \AA}$, $\tau_N = 10^{-8} \text{ sec}$ (the value for the sodium D line), and $f = 1/2$ which assumes the resonance line is saturated and the ground and excited states are equally degenerate.²¹

The resultant force is $6.6 \times 10^{-15} \text{ dynes}$ and the initial acceleration for the case of the sodium atom would be $1.76 \times 10^5 \text{ G's}$. The power level to produce saturation in this case is $.019 \text{ W/cm}^2$ (as will be shown below).

Equation III-9 can be written in an alternate form by noting that the natural linewidth of the resonance line is related to the lifetime of the excited state, i.e. $\Delta\nu = 1/2\pi\tau_N$. If we define a quality factor for the resonance line as $Q = \nu_0/\Delta\nu$, we can rewrite the force equation as

$$F = \frac{2\pi h \nu_0^2}{c \mathcal{F}} f$$

III-10

where c is the speed of light. If we assume that the quality factor of most resonance lines is approximately independent of the resonance frequency, Q , this equation makes it clear that the force resulting from atomic resonant absorption is proportional to the second power of the resonant frequency. Consequently, one should not expect significant forces (or accelerations) from resonant absorption in the microwave frequency range.

Ashkin makes the observation in his paper ²¹ that, depending on the application, doppler shifts of the atoms as they gain speed may limit the achievable acceleration by causing the atoms to move off of resonance. One solution to this, as Ashkin notes, is to power broaden the resonance line so that the effective linewidth is increased. A broader resonance line may also make frequency control of the laser supplying the resonant radiation unnecessary. It then becomes relevant to know how much power is needed to achieve saturation since broadening ensues at powers greater than this. To do this calculation we need to know the scattering cross section for resonance absorption. The optical theorem states ²²

$$C_{abs} = 4\pi k \cdot \text{Re}(\alpha) \quad \text{III-11}$$

where C_{abs} is the scattering cross section for absorption, k is the wavevector of the incoming radiation $k = 2\pi/\lambda$, Re stands for the real part of, $i = \sqrt{-1}$, and α is the polarizability. Using a simple classical model for damped harmonic motion of an electron, the polarizability is given as ²³

$$\alpha = \frac{e^2}{4\pi^2 M} \frac{(\nu_0^2 - \nu^2) - i\nu\Delta\nu}{(\nu_0^2 - \nu^2)^2 + (\nu\Delta\nu)^2} \quad \text{III-12}$$

where ν_0 is the resonance frequency, M is the electron mass, e is the electron charge, and $\Delta\nu$ is the linewidth. The cross section for absorption is thus

$$C_{abs} = \frac{2e^2 \nu^2 \Delta\nu}{Mc ((\nu_0^2 - \nu^2)^2 + (\nu\Delta\nu)^2)} \quad \text{III-13}$$

Evaluating this on resonance by taking $\Delta\nu = \gamma/2\pi = 1.59 \times 10^7 \text{ Hz}$ yields $C_{abs} = 1.06 \times 10^{-28} \text{ m}^2$. The force on the atom based on the absorption cross section is given as

$$F = C_{abs} \frac{W}{c} \quad \text{III-14}$$

Equating this equation to Eqn. III-9 and then solving for the intensity W gives $W \sim 1.9 \times 10^5 \text{ ergs/cm}^2$ or $.019 \text{ Watts/cm}^2$ to saturate the line. This intensity is sufficiently low that saturation can easily be achieved and significant power broadening can be accomplished with realizable laser sources. Ashkin discusses the size of the broadening as a function of power level in his paper.²¹

The real limitation to the use of resonant radiation pressure lies in the requirement that a laser source be available at precisely the resonant frequency of interest. However, strides are being made in the development of tunable lasers so that this interesting technique may lend itself to applications in the near future.

Conclusion

This completes the section on radiation pressure. However, since the size of the acceleration that can be produced is directly related to the intensity of the "light" source, some comment as to the power levels that can be achieved is in order. For the ultraviolet Helium Cadmium laser (.325 μ) only 10 mW²⁴ total power is possible. The Helium Neon laser is capable of 100 mW²⁴. The CO₂ laser is capable of extremely high powers, 10 kW or greater, but these are only for very expensive, very large lasers. Considering a laser 3 feet long as a maximum size, 100 W²⁵ is a reasonable power level that can be expected. All of these values are for CW operation. It should be noted that very high intensities can be achieved by focussing the laser and thus placing the total power available in a small area. The theoretical limit to this focusing is an area that is equal to the square of the wavelength.

IV. SOUND PRESSURE

Our investigation has been limited to compressional waves in gases and liquids, and the effects of such waves on particles suspended in the medium.

Particles may be moved by two effects: radiation pressure and/or acoustic streaming.

Radiation Pressure

The DC radiation pressure is described by an equation similar in form to the expression for radiation pressure in an electromagnetic wave, that is, the pressure is equal in value to the energy density of the wave.²⁶ The net DC force produced on an obstacle in the medium carrying the sound wave is equal to the effective cross section area ratio for momentum transfer times the actual cross section area of the particle times the energy density of the sonic wave. As in electromagnetic radiation the effective cross section area of the particle is much smaller than the actual cross section for particle sizes smaller than a wavelength, so that little force results in that regime.

Now in gases, the maximum sonic frequency which can be transmitted without excessive attenuation due to the medium itself is in the order of 10^6 Hz, corresponding to a wavelength of 10^{-2} to 10^{-1} cm.²⁷ Thus, manipulation of small particles in a gaseous medium using sonic radiation pressure appears to be impractical, since micron sized particles would be much smaller than a wavelength of sound.

In a liquid medium, however, frequencies up to the order of 10^9 Hz can be propagated,²⁷ corresponding to wavelengths of 1 to 10 microns, so that smaller particles can be moved in a liquid medium. We have not been able to pursue this use of sound radiation pressure in detail, because of limits of time and money, but it is worthy of further investigation, especially as to the characteristics of practical sources of energy at such frequencies, and the sonic reflective or absorptive characteristics of particular substances.

Acoustic Streaming

Acoustic streaming is another sonic effect which we have not been able to pursue in detail during this study, but which shows interesting possibilities for manipulation of small particles. Acoustic streaming is a DC, convective type of motion of the medium which occurs under certain circumstances with sonic radiation. Particles suspended in the medium will experience degrees of motion related to their size, thus forming a possible means of sorting.

One type of acoustic streaming is caused by the presence of solid boundaries such as the walls in a resonant chamber containing a standing wave.

The streaming or vortical motion of the medium results from the fact that no slip motion can occur at the walls. A well known example of this phenomenon

is the "K" which fine sand or lycopodium powder is shaken into a standing wave visible. When the tube is energized, the powder is moved from the interaction of the standing wave in the tube causes the powder to be transported and deposited out of the nodes of the standing wave.

Lord Rayleigh²⁸ and Bessel²⁹, among others, solved this problem for the particular case of a standing wave of sound existing in a waveguide formed by a pair of parallel plates terminated at one end by a reflecting wall so as to produce a standing wave. They derived equations for the net horizontal and vertical DC streaming motions of the particles of the medium during steady state excitation.

Outside the motionless thin film immediately adjacent to the walls, the net horizontal velocity, u_2 , is given by

$$u_2 = - \frac{3u_0^2 \sin 2Kx}{16c} \left\{ 1 - \frac{3(y_1 - y)^2}{y_1^2} \right\} \quad (1)$$

and the net DC vertical velocity, perpendicular to the walls, is given by

$$v_2 = \frac{-3u_0^2 2K \sin 2Kx}{16c} \left\{ y_1 - y - \frac{(y_1 - y)^3}{y_1^2} \right\} \quad (2)$$

where

- y is the distance from one wall to the point under consideration
- y_1 is the distance from one wall to the mid-plane of symmetry
- u_0 is the maximum value of the instantaneous horizontal velocity
- K is the wave number of the sound wave, i.e. $K = (2\pi/\lambda)$
- x is the horizontal distance from the sound source
- c is the velocity of sound in the medium

It is seen that these motions describe a series of vortices periodic with respect to x in the distance $\lambda/2$. The vertical velocity, v_2 , is negative, that is, towards the walls, at the loops, and positive, or away from the wall, at the nodes, vanishing in each case both at the wall, $y = 0$, and at the plane of symmetry, $y = y_1$. The equation for u_2 , the average horizontal velocity, shows that it is zero at both the nodes and the loops, and has maximum positive and negative values half-way between the nodes and the loops.

The nature of the vortices explains the behavior of the Kundt's tube. It is seen that particles suspended in the medium will be carried horizontally to, and deposited out at, the nodes.

In Figure 1, AB represents the wall and CD the plane of symmetry, and the directions of motions in the vortices are indicated by the arrows.

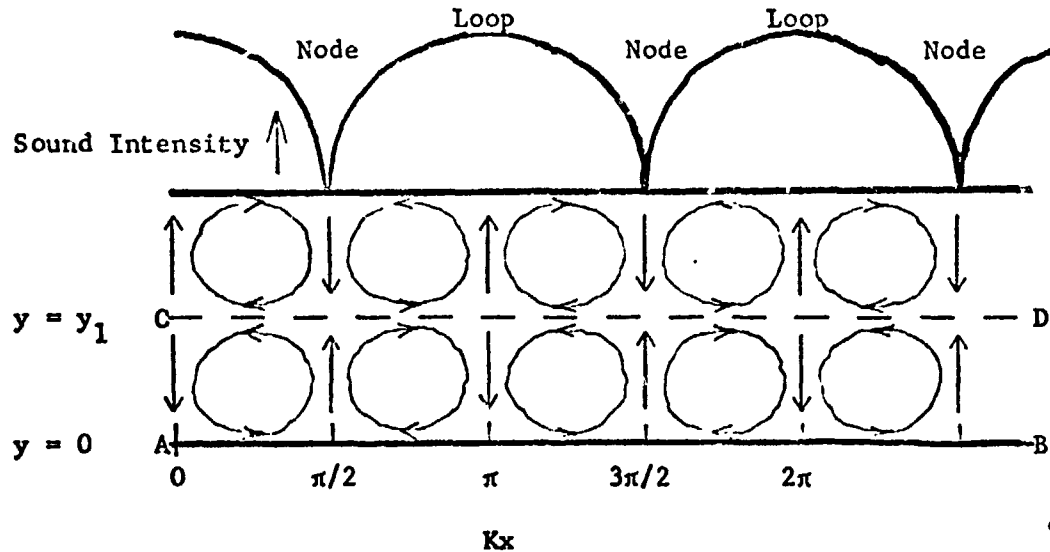


Figure 1 - Streaming Vortices in a Chamber Containing a Standing Wave

We have in a preliminary way extended Rayleigh's analysis to explore the behavior of the vortices in the frequency domain and to determine the effect of the streaming motions of the medium on particles of various sizes immersed in the medium.

One can assume a transient situation in which the sound is turned on and a short time later the streaming motions of the medium reach steady state velocity. At this early time the suspended particles, which are assumed to be very much larger than the particles of the medium, are assumed to have reached only a small fraction of the velocity of the medium. We can then calculate the force on these particles using Stoke's law which states the proportionality between drag force and the product of the differential velocity, viscosity of the medium, and particle radius. Dividing the drag force by the mass of the particle yields a value for initial acceleration of the particles.

Initial accelerations computed in this way do not of course show directly what can be achieved in a particle sorting device based on these principles, but the order of magnitude of the initial acceleration produced and its variation with particle parameters gives a general indication of the potentialities of such devices.

The results of this preliminary analyses yield the following conclusions:

- 1) At low frequencies of excitation the particles would tend to be deposited at the nodes in relatively narrow bands.
- 2) At high frequencies the deposition would still concentrate at the nodes but much more diffusely than at low frequencies.
- 3) The initial accelerations are greater at the higher frequencies.
- 4) The acceleration produced is inversely proportional to the product of the density of the particle and the square of its radius.
- 5) In a practical case we assumed water droplets of one micron radius in air in a parallel plate channel containing a standing wave with acoustic sound level of 126 dB above 0.002 dynes/cm². At low frequencies the maximum initial axial acceleration was calculated to be 17 cm/sec² or ~0.017 G, and the maximum initial vertical acceleration was found to be 39 cm/sec², or 0.039 G. At high sonic frequencies the maximum initial axial acceleration remains about the same, but the vertical components increase with frequency.

The preliminary results indicate that acoustic streaming will produce significant motion of particles. Further work is needed to confirm what has already been done, and to work out in more detail the particle motions due to streaming forces in a viscous medium.

Other Acoustic Forces

Three other acoustic forces are listed by Hueter and Bolt³⁰, but have not been investigated in this program. They are mentioned here as a guide to possible future work. They are as follows:

Type of Force	Cause	Drag Coefficient	Direction of Force
"Average Stokes"	Temperature dependence of viscosity	$\frac{3(\gamma - 3)\eta}{\pi \rho}$	Toward Source
"Oseen"	Wave distortion	$\frac{3}{\pi} C_2 \sin \phi$	Depends on phase
"Bernoulli"	Hydrodynamic Flow	$\frac{3}{2} \frac{\eta_1 \eta_2}{d^4}$	Attraction between 2 particles

The symbols used are defined as follows:

- r = particle radius
- d = distance between two adjacent particles
- γ = specific heat ratio
- η = viscosity coefficient
- ρ = density of the medium
- $C_2 = u_2/u_0$ = fractional 2nd harmonic content
- ϕ = phase shift between fundamental and 2nd harmonic of distorted sound wave

Of these forces, the force of attraction between adjacent particles is of interest because it can be used to produce agglomeration of particles.

V. PARTICULATE RADIATION

We consider particulate radiation to denote streams of elementary particles such as electrons, ions, or uncharged gas molecules.

We have considered the case of an electron beam impinging on an aggregation of particles and have calculated the forces and accelerations produced by relatively low energy electrons whose mass is approximately equal to the rest mass. We will assume for simplicity that the electron is totally absorbed by the particle it strikes so that it gives up all its energy and momentum to the particle.

The derivation of the force equation is quite simple, and it proceeds as follows:

The force on the particle is just the momentum of each electron times the number of electrons captured per second

$$F = [m_e v_e] [6.28 \times 10^{18} J] [\pi a^2] \quad \text{I-1}$$

momentum
#electrons
capture
per electron
per sec cm²
area of

particle

For velocities much less than the velocity of light the electron velocity is given by

$$v_e = 5.92 \sqrt{V} \times 10^7 \text{ cm/sec} \quad \text{I-2}$$

where V is the potential difference in volts through which the electron was accelerated.

The middle term arises from the fact that there are 6.28×10^{18} electrons/second per ampere of current. J is the current density in amperes/cm².

Putting in values for m_e and v_e we get for the force on a spherical particle of radius a cm

$$F = 1.06 J a^2 \sqrt{V} \text{ dynes} \quad \text{I-3}$$

The acceleration produced in a vacuum is obtained by dividing both sides of the equation by the particle mass, $(4/3)\pi a^3 \rho$, yielding

$$\text{Accel.} = \frac{F}{M} = 0.254 \frac{J \sqrt{V}}{\rho a} \text{ cm/sec}^2 \quad \text{I-4}$$

In a specific case we assume an electron beam of 100 volts energy and 10 milliamperes/cm² current density, impinging on particles of 1μ radius and 1 gram/cm³ density in an evacuated chamber. Putting these values into the acceleration equation, we find a large acceleration of 254 cm/sec² or 0.259 G for this example.

Interaction Chamber

The above calculations assumed an idealized situation in which electrons continuously strike particles and give up all their energy and momentum to them. In a practical case, however, there are several characteristics of the electron-particle collisions which must be considered before it can be determined whether the idealized model can be practically realized. Let us consider the case of the evacuated chamber first and determine what happens when the electron beam is turned on. We will assume an electron energy in the order of 10 volts, low enough that secondary electron emission will not complicate the problem. Initially the electrons will be absorbed by the particles pretty much in accordance with the simple model on which the force equation was derived. As time goes on the particles will become negatively charged, and no further electrons will be absorbed when the potential of the particles reaches the energy of the beam.

Nevertheless, even though no further electrons are absorbed by the particles after charge equilibrium is reached, additional electrons will be stopped or scattered by the potential field of the particles, so that momentum should continue to be transferred to them, and the effective cross section of the particles should continue to be the same order of magnitude as their actual cross section area. But then the problem becomes more complicated because potential differences may exist between the walls of the container and the particles which will produce additional forces on the particles.

It was beyond the scope of this study to work out the kinds of interaction chambers which might be required to utilize the force created by an electron beam impinging on particles. The charges produced on the particles certainly do complicate the practical problem of utilization devices compared to the use of the other force fields considered, which do not produce charges on the particles.

It should also be noted that it may be difficult to use an electron beam to manipulate particles in a viscous medium such as air because of the large absorption or scattering of the beam by the gas molecules themselves. For example, the attenuation constant for a 100 volt electron beam entering neon gas (which has a relatively low collision cross section for electrons) at 1 torr pressure is 10 per cm, so that the beam would be reduced to 1/e of its initial intensity in 0.1 cm.³¹

VI. ELECTROSTATIC FORCES

Electrostatic forces consist of coulomb forces, i.e., forces on charged particles in an electric field, or electrophoretic forces, i.e., forces on small charged particles in colloidal solutions, or dielectrophoretic forces, i.e., forces on uncharged particles in an electric field.

We have not treated coulomb forces, because in general, they are large forces, their characteristics are widely known and utilized in electrostatic precipitators, and they are not generally capable of fine control because of the uncertainty of the amount of charge put on the particles by corona discharge. We have not considered electrophoretic forces because we understand that NASA already has a program of evaluation of electrophoretic effects in the zero gravity environment.

Dielectrophoresis

Dielectrophoresis is the name given to the production of forces on uncharged particles in a non-uniform electric field resulting from the electric polarization of the particle. The polarization is caused by shifts in the electron clouds of the atoms produced by the electric field so that each atom acquires a dipole moment. If the field is uniform, equal and opposite forces are produced on the induced charges of the dipole, and no net force results on the particle. If the field is non-uniform, however, the forces on the dipole are not equal and opposite, and a weak net force exists on the particle. Dielectrophoretic forces are predictable and reproducible, since they originate from fundamental properties of the particle and of the field.

The derivation of the expression for the magnitude of the dielectrophoretic force is given in advanced texts on electromagnetic theory, such as Stratton's Electromagnetic Theory, 1st edition 1941 published by McGraw Hill. The derivation will be sketched out here, and then some typical examples of the magnitude of acceleration which can be produced on various substances in a simple coaxial structure with a potential difference applied between the coaxial elements will be given.

Stratton gives the change in energy resulting from introducing a body of permittivity ϵ_2 into a uniform medium of permittivity ϵ_1 as ³²

$$U = \frac{1}{2} \int_{Vol} (\epsilon_1 - \epsilon_2) \vec{E} \cdot \vec{E}_0 dV \quad \text{VI-1}$$

where \vec{E} is the electric field vector inside the body, and \vec{E}_0 is the electric field vector in the neighborhood of the particle before it was brought into the field, i.e., the applied field. If the body is spherical, Stratton shows the relation between \vec{E} and \vec{E}_0 for a uniform field to be ³³

$$\vec{E} = \frac{3\epsilon_1}{\epsilon_2 + 2\epsilon_1} \vec{E}_0 \quad \text{VI-2}$$

Looking ahead, one notes that the field must be non-uniform in order for there to be a force, so that there is a question as to the validity of using Eq. VI-2 for this case. It can be shown, however, that Eq. VI-2 gives the zero order term in the electric field for the non-uniform case, and also becomes a reasonable approximation to the total field if the variation in the field is small over the volume of the sphere. If we now substitute the value of E given by Eq. VI-2 into the integral expression for the energy, given by Eq. VI-1, we note that the integrand is spatially varying, thus complicating the evaluation of the integral. Nevertheless, the integral can be evaluated by taking the electric field to be constant, using the value at the center of the sphere as the mean value. This will be a good approximation to the integral if the variation in field is small over the volume of the sphere.

The resulting value of the change in energy due to introducing the sphere turns out to be

$$U = \left(\frac{\epsilon_1}{\epsilon_2 - 2\epsilon_1} \right) 2\pi a^3 E_0^2 \quad \text{VI-3}$$

where a is the radius of the sphere.

The force acting on the sphere is the negative of the gradient of the energy change in the direction of the force. Thus

$$\vec{F} = -\nabla U \quad \text{VI-4}$$

and performing the indicated differentiation, and substituting $K_1\epsilon_0$ for ϵ_1 and $K_2\epsilon_0$ for ϵ_2 , where K_1 and K_2 are the relative dielectric constants of the medium and the particle respectively, and ϵ_0 is the permittivity of a vacuum, we find that the expression for the force on the particle is

$$\vec{F} = 4\pi a^3 \epsilon_0 K_1 \frac{(K_2 - K_1)}{(K_2 + 2K_1)} \vec{E}_0 \cdot \nabla \vec{E}_0 \quad \text{VI-5}$$

This expression is in MKS units. The force is in newtons (1 newton = 10^5 dynes); the particle radius a is in meters, the electric field, E_0 , is in volts per meter, and ϵ_0 is the permittivity of a vacuum = 8.855×10^{-12} farads/meter.

The main assumptions on which the validity of the above analysis depends are that (1), the particles are small compared to the distance in which the electric field changes appreciably, and that (2), the number of particles per unit volume is small enough that the presence of the particles does not appreciably affect the pattern of the applied field. For small particles dispersed in a reasonably sized interaction chamber, these assumptions should be met.

Examples of Dielectrophoretic Force

Let us consider the case of a particle-field interaction chamber consisting of two coaxial electrodes of radius R_1 and R_2 meters with the particles located in the annular space between the electrodes, and a potential difference of ϕ_0 volts applied to the electrodes. Such a configuration yields a highly non-uniform radial electric field between the electrodes. It can be shown that in this configuration the product of $E_{on} \nabla E_{on}$, where r is in the radial direction in this case, is

$$E_{on} \nabla E_{on} = \frac{\phi_0^2}{\pi^3 [\ln R_1/R_2]^2} \quad \text{VI-6}$$

where r is the radial distance outward from the axis of the cylinders.

Thus in this particular configuration the force on the particle is given by:

$$F_n = 4\pi a^3 \epsilon_0 K_1 \frac{(K_2 - K_1)}{(K_2 + 2K_1)} \left[\frac{\phi_0^2}{\pi^3 [\ln R_1/R_2]^2} \right] \quad \text{VI-7}$$

and the acceleration in a vacuum by

$$\text{Accel}_n = \frac{F_n}{M} = 3\epsilon_0 \frac{(K_2 - K_1)}{(K_2 + 2K_1)} \frac{1}{\rho} \frac{\phi_0^2}{\pi^3 [\ln R_1/R_2]^2} \quad \text{VI-8}$$

where ρ is the density of the particle in Kg/meter³.

Note that the acceleration in a vacuum produced by the dielectrophoretic force is dependent on the dielectric constant of the particle and its density, but is independent of its size.

If we now assume that the applied potential, ϕ_0 is 10,000 volts, that R_1 is 1 cm and R_2 is 2 cm, that $x = R_1 = 1$ cm (meaning that the particle is located near the surface of the inner electrode), we find accelerations expressed in units of g as follows for a number of substances:

Table VI-1

Substance	Density, in Kgr/meter ³	Dielectric Constant, K_2 (dimension- less)	$\frac{1}{\rho} \frac{(K_2-1)}{(K_2+2)}$ Clausius- Mosotti Constant	Acceleration produced at the center of a coaxial chamber with $R_1 = 1$ cm, $R_2 = 2$ cm, and 10,000 volts applied
Water	1.0×10^3	81.0	0.97×10^{-3}	-0.55 g
Oil	0.8×10^3	2.2	0.36×10^{-3}	-0.21 g
Paper	$\sim 0.8 \times 10^3$	~ 2.5	0.42×10^{-3}	-0.24 g
Boro- Silicate Glass	2.24×10^3	4.86	0.25×10^{-3}	-0.144 g
Diamond	3.51×10^3	16.5	0.24×10^{-3}	-0.135 g
Mica	3.0×10^3	6.0	0.21×10^{-3}	-0.119 g
Sulphur	2.07×10^3	4.2	0.25×10^{-3}	-0.142 g
Dry Wood	$\sim 0.5 \times 10^3$	~ 6.0	1.25×10^{-3}	-0.712 g

If we assume that a lower limit of manipulation would occur when the energy added to the particle by the dielectrophoretic force acting on the particle all the way from the outer electrode to the inner electrode is equal to its thermal energy, we find an estimate of the minimum size of manipulatable particles to be about 0.06 microns for the particular interaction chamber described above.

Summary of Dielectrophoresis Effects

In summary, the accelerations produced by dielectrophoresis are very substantial, being in the order of 0.1 to 1 g, they are independent of particle size, they are inversely proportional to the density of the particle, and they are a function of the dielectric constant of the particle. Particles down to about 0.05 microns radius should be manipulatable before random thermal energy interferes.

VII. MAGNETIC FORCES

We are all familiar with the very large forces exerted on ferromagnetic substances by magnetic fields generated either by electromagnets or by permanent magnets. These forces may be exerted on particles of ferromagnetic substances which have no initial magnetic poles as well as on permanent magnets. In the former case, the magnetic field causes the highly polarizable domains of the magnetic material to line up with the field. If the magnetic field is uniform in the direction of the field, equal and opposite forces are applied to the induced magnetic poles in the particle, and no net force on the particle results.

If, on the other hand, the magnetic field is non-uniform in the direction of the field, then the forces on the induced poles are not equal, and there is a net force on the particle which urges it in the direction of the stronger field. The direction and magnitude of the force are independent of the polarity of the field, so that a net force is produced in AC as well as in DC fields.

The derivation of the force equation for materials in a magnetic field follows the same steps as the derivation for the dielectrophoretic force described in the previous section, and equations VI-1, 2, 3, 4, 5 and 6, when changed to magnetic units, describe the steps in the derivation of the magnetic force.

For the magnetic case, the dielectric constant, K_2 , of the particle is replaced by the magnetic permeability, μ of the particle, and the dielectric constant, K_1 of the medium, is replaced by unity, since it is assumed that the medium will be non-magnetic. The electric fields, \vec{E} and \vec{E}_0 , are replaced by magnetic fields \vec{H} and \vec{H}_0 . The resultant equation for the magnetic force on a spherical particle is^o

$$F = \frac{\mu - 1}{\mu + 2} a^3 \vec{H}_0 \cdot \nabla \vec{H}_0 \text{ dynes} \quad \text{VII-1}$$

where

μ is the magnetic permeability of the particle
 a is the radius of the particle in cm
 \vec{H}_0 is the field strength in oersteds
 $\frac{\vec{H}_0 \cdot \nabla \vec{H}_0}{|\vec{H}_0|}$ is the directional derivative of the field in the direction of the field in oersteds/cm

We note that this force is proportional to the volume of the particle, whereas electromagnetic radiation pressure, sound pressure, and electron beams all produced forces proportional to the area of the particles.

Note that this is in cgs units, while dielectrophoretic force is in mks units. In the following, we will take the case where the field and its derivative are both in the x direction.

If we divide both sides of the equation by the mass of the particle, where $M = (4/3)\pi a^3$, we get the expression for the acceleration produced in a vacuum, or the initial acceleration produced in a viscous fluid,

$$\text{Accel}_x = \frac{F_x}{M} = 0.238 \frac{1(\mu - 1)}{\rho(\mu + 2)} H_{ox} \frac{dH_{ox}}{dx} \text{ cm/sec}^2 \quad \text{VII-2}$$

and we see that this is dependent only on the density and the magnetic properties of the particle, and not on its size.

Forces on Ferromagnetic Particles

Now if μ is much greater than 1, as is the case for ferromagnetic substances, the fraction $(\mu-1)/(\mu+2)$ is practical unity, so that the acceleration equation simplifies to

$$\text{Accel}_x = 0.238 \frac{H_{ox}}{\rho} \frac{dH_{ox}}{dx} \text{ cm/sec}^2 \text{ for ferromagnetic particles} \quad \text{VII-3}$$

It is possible in magnetic equipment to develop a product of $H_{ox} (dH_{ox}/dx)$ of magnitude 10^7 oersteds²/cm, and putting this number along with a typical ferromagnetic material density of 8 grams/cm³ into the acceleration equation, we get

$$\text{Accel (ferromagnetic)} = 300,000 \text{ cm/sec}^2 \quad \text{VII-4}$$

$$\approx 300 \text{ G}$$

This is a very large effect indeed, and can be used, for example, to remove ferromagnetic particles from other particles with a good separation efficiency.

Its magnitude is sufficient to warrant a look at the limiting velocities in viscous media such as air or water. To determine the limiting velocity, we equate the driving force as derived above to the drag force as given by Stoke's Law. Again assuming a practical value of 10^7 oersteds²/cm for $H_{ox} (dH_{ox}/dx)$ as in the previous example, we find

$$\underbrace{10^7 a^3}_{\text{driving force}} = \underbrace{6\pi\eta av}_{\text{drag force}} \quad \text{VII-5}$$

$$v = 5.3 \times 10^5 \frac{a^2}{\eta} \text{ cm/sec} \quad \text{VII-6}$$

where η is the viscosity of the fluid in poises. For a ferromagnetic particle of 1 micron radius the limiting velocities turn out to be

$$v_{lim} \text{ (air at NTP)} = 30 \text{ cm/sec} \quad \text{VII-7}$$

$$v_{lim} \text{ (water)} = 30 \text{ cm/sec} \quad \text{VII-8}$$

These velocities are very substantial, even in water. In air they should produce usable effects even for magnetic particles smaller than 1 micron in radius.

Forces on Diamagnetic and Paramagnetic Particles

Most non-ferromagnetic materials have permeabilities which differ slightly from unity, typically by a few parts per million. The permeability may be either slightly greater than unity, as in paramagnetic materials, or slightly less than unity, as in diamagnetic materials.

The force equation for such substances can be simplified because the factor $(\mu+2)$ in the denominator is very nearly equal to 3 for diamagnetic and paramagnetic substances. In this case the force equation becomes

$$F_x = \frac{\mu - 1}{3} a^3 H_{ox} \frac{dH_{ox}}{dx} \text{ dynes} \quad \text{VII-9}$$

and the acceleration equation for the practical case of $H_{ox} (dH_{ox}/dx) = 10^7$ oersteds²/cm becomes

$$\text{Accel}_x = \frac{F_x}{M} = 8 \times 10^{+5} \frac{\mu - 1}{\rho} \text{ cm/sec}^2 \quad \text{VII-10}$$

The factor $(\mu-1)/\rho$ is called the "specific magnetic susceptibility," and values for this parameter for many substances are listed in the Handbook of Chemistry and Physics, 42nd Edition, pp. 1647 to 2657. Specific susceptibilities run from about $+10^{-4}$ to -10^{-6} for a wide range of elements and compounds. Taking a few examples, we find values as follows:

Substance	Specific Magnetic Susceptibility	Acceleration Produced in a Practical Non-Linear Field where $H_{ox} (dH_{ox}/dx) = 10^7$ oersteds
Mercury (liquid)	-0.15×10^{-6}	-0.12 cm/sec^2
Arsenic	-0.31×10^{-6}	-0.25 cm/sec^2
Wood	$-0.3 \text{ to } -0.7 \times 10^{-6}$	$-0.24 \text{ to } -0.56 \text{ cm/sec}^2$
Aluminum	$+0.6 \times 10^{-6}$	$+0.48 \text{ cm/sec}^2$
Air at NTP	$+24 \times 10^{-6}$	$+19 \text{ cm/sec}^2$ (but this is too low a velocity to compete with random thermal velocity of the molecules)

In summary we can say that the forces produced on ferromagnetic particles by application of a non-uniform AC or DC magnetic field are very large. They will produce large accelerations in a vacuum which are independent of the size of the particles. Even in viscous media such as air or water, appreciable values of limiting velocity can be obtained.

The analysis given here applies only to low frequency AC fields where the curl terms in Maxwell's equations are negligably small. This will generally be true for excitation frequencies in the audio frequency range.

Although the forces on non-ferromagnetic materials are orders of magnitude less than the forces on ferromagnetic materials, they may be sufficient to produce useful sorting effects in an evacuated chamber.

VIII. SUMMARY

It was the purpose of this study to obtain quantitative understanding of a series of weak forces acting on small particles. The forces considered were those derived from light, heat, and microwaves (electromagnetic radiation pressure), electric interactions, magnetic interactions, particulate interactions, and sound. Throughout the body of this report, we have attempted to give a physical understanding of the forces considered as well as formulae which express how the size of the particular force depends on the physical and electrical properties of the particle.

In the second section we have evaluated the drift velocity in a viscous fluid as a function of initial acceleration in addition to considering the competing effects of thermal random motion. Out of all of this comes a means of selectively sorting or moving particles by choosing a force system and/or environment such that the particle of interest reacts uniquely.

The table below will serve as a summary of the forces considered and a demonstration of how the initial acceleration, drift velocity, and ultimate particle density distribution (ΔX is the distance in which approximately 70 percent of the particles will be confined) is affected by particle, input, and environmental parameters.

The dielectrophoretic and magnetic forces are identical as to their dependence on particle size and are unique in that the initial acceleration does not depend on particle size. The dependence of response parameters (acceleration, drift velocity, ΔX) for the electromagnetic radiation force is probably the most unique of all the forces considered because of the strong dependence of the efficiency factor Q_{pr} on particle size. Consequently, radiation pressure offers the best possibilities for selectively moving particles on the basis of particle size alone. Sound pressure is not included in the summary because the force is quite complex and not amenable to a short summary. In all of the cases in the table, the drift velocity and the parameter ΔX are independent of particle density since the particle forces are independent of mass.

It is difficult to summarize the sizes of the acceleration, drift velocity, and confinement distance because they are very dependent on the excitation levels used as well as the particle parameters. However, in order to convey some indication of the size of effect to be expected, values which are typical in a gross sense are given for the response parameters for each of the forces in the table. However, for complete appraisal of the sizes of the response parameters that can be achieved, particularly for the case of radiation pressure, reference should be made to the appropriate section of this report.

The reader should note that in the table the expressions for acceleration, drift velocity, and confinement distance under row 2, Magnetic Force, are given in mks units. The equations used to describe magnetic effects in the body of the text are given in cgs units.

SUMMARY OF SMALL FORCES STUDY

Acceleration in a vacuum

1. Dielectrophoresis

$$\vec{A}_{cc} = \frac{1}{\rho} \left(\frac{3\epsilon_1(\epsilon_2 - \epsilon_1)}{(\epsilon_2 + 2\epsilon_1)} \right) \vec{E}_0 \cdot \nabla \vec{E}_0$$

$$\approx 0.3 \text{ G's} \quad (\rho = 1.9 \text{ m/cm}^3)$$

2. Magnetic Force

$$\vec{A}_{cc} = \frac{1}{\rho} \left(\frac{3\mu_0(\mu - \mu_0)}{(\mu + 2\mu_0)} \right) \vec{H}_0 \cdot \nabla \vec{H}_0$$

$$\approx 300 \text{ G's} \quad (\text{magnetic}, \rho = 8.9 \text{ m/cm}^3)$$

$$\approx .001 \text{ G's} \quad (\text{non-magnetic}, \rho = 1.9 \text{ m/cm}^3)$$

3. E.M. Radiation Pressure

$$A_{cc} = \frac{Q_{pr}(\chi, m)}{\rho a} \left(\frac{3}{4} \frac{W}{c} \right); \chi = \frac{2\pi a}{\lambda}$$

$$\approx 3 \text{ G's} \quad (a = .1 \mu, W = 100 \text{ W/cm}^2, \rho = 1.9 \text{ m/cm}^3)$$

4. Resonant Radiation Pressure

$$A_{cc} = \frac{\pi \hbar \nu_0^2}{M C \xi}; \xi = \nu_0 / \Delta \nu$$

$$\approx 10^5 \text{ G's} \quad (\text{atomic size})$$

5. Particulate Radiation

$$A_{cc} = \frac{1.2 J V^{1/2}}{\rho a}$$

$$\approx 0.3 \text{ G's} \quad (a = 1 \mu, J = 10 \text{ ma/cm}^2, \rho = 1.9 \text{ m/cm}^3, V = 100 \text{ volts})$$

Drift velocity in a viscous fluid

$$\vec{V} = \frac{Q^2}{\eta} \left(\frac{2}{3} \epsilon_1 \frac{(\epsilon_2 - \epsilon_1)}{(\epsilon_2 + 2\epsilon_1)} \right) \vec{E}_0 \cdot \nabla \vec{E}_0$$

$$\approx 0.03 \text{ cm/sec} \quad (a = 1 \mu, \text{air})$$

$$\vec{V} = \frac{Q^2}{\eta} \left(\frac{2}{3} \mu_0 \frac{(\mu - \mu_0)}{(\mu + 2\mu_0)} \right) \vec{H}_0 \cdot \nabla \vec{H}_0$$

$$\approx 30 \text{ cm/sec} \quad (a = 1 \mu, \text{air})$$

$$\approx 1 \times 10^{-5} \text{ cm/sec} \quad (a = 1 \mu, \text{air})$$

$$V = \frac{Q_{pr} a}{\eta} \left(\frac{1}{6} \frac{W}{c} \right)$$

$$\approx 3 \times 10^{-7} \text{ cm/sec} \quad (a = .1 \mu, \text{air})$$

$$V = \frac{1}{\eta a} \left(\frac{1}{6} \frac{\hbar \nu_0^2}{c \xi} \right)$$

$$\approx 1 \times 10^{-7} \text{ cm/sec} \quad (\text{air})$$

$$V = \frac{Q}{\eta} 0.27 J V^{1/2}$$

$$\approx 0.03 \text{ cm/sec} \quad (a = 1 \mu, \text{air})$$

Confinement distance

$$\Delta x = \frac{\kappa T / a^3}{\left(\frac{4 \pi \epsilon_1 (\epsilon_2 - \epsilon_1)}{(\epsilon_2 + 2\epsilon_1)} \right) \left(\frac{\vec{E}_0 \cdot \nabla \vec{E}_0}{a} \right)}$$

$$< a \quad (a = 1 \mu)$$

$$\Delta x = \frac{\kappa T / a^3}{\left(\frac{4 \pi \mu_0 (\mu - \mu_0)}{(\mu + 2\mu_0)} \right) \left(\frac{\vec{H}_0 \cdot \nabla \vec{H}_0}{a} \right)}$$

$$< a \quad (a = 1 \mu)$$

$$\approx 100 \mu \quad (a = 1 \mu)$$

$$\Delta x = \frac{\kappa T / (Q_{pr} a)}{\pi W / c}$$

$$\approx 30 \mu \quad (a = .1 \mu)$$

$$\Delta x = \frac{\kappa T}{\pi \hbar \nu_0^2 / c \xi}$$

$$\approx 1 \text{ cm}$$

$$\Delta x = \frac{\kappa T / a^2}{1.6 J V^{1/2}}$$

$$< a \quad (a = 1 \mu)$$

a = radius of particle

ρ = density of particle

η = viscosity of fluid

k = Boltzmann's constant

T = absolute temperature

ϵ_2 = permittivity of particle

ϵ_1 = permittivity of background

μ = permeability of particle

μ_0 = permeability of free space

E_0 = electric field

H_0 = magnetic field

$\frac{\vec{E} \cdot \nabla \vec{E}}{|\vec{E}|} =$ electric field directional derivative

$\frac{\vec{H} \cdot \nabla \vec{H}}{|\vec{H}|} =$ magnetic field directional derivative

$Q_{pr} =$ efficiency factor for radiation pressure

λ = wavelength of E.M. radiation

$W =$ intensity of E.M. radiation $\frac{(\text{watts})}{\text{cm}^2}$

c = speed of light

m = refractive index of particle

h = Planck's constant

ν_0 = resonant frequency of atom

M = particle mass

$F =$ quality factor of resonant line = $\nu_0/\Delta\nu$

$J =$ current density

$V =$ accelerating voltage of electrons

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